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**APPLICATION OF REGRESSION ANALYSIS TECHNIQUES  
TO REFRACTORY COATING MEASUREMENT  
EXPERIMENTS**

By Bobby G. Junkin  
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April 27, 1971

**NASA**

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## DEFINITION OF SYMBOLS

Symbol	Definition
$Y^c$	Computed response variable (dependent)
$Y^o$	Observed response variable
$\sigma_{Y^c}$	Standard deviation of the response variable
$b_0, b_1, b_2, \dots, b_p$	Regression coefficients
$Z_1, Z_2, \dots, Z_p$	Independent variables
$\bar{Z}_1, \bar{Z}_2, \dots, \bar{Z}_p$	Means of the $p$ independent variables
$\sigma_{b_0}, \sigma_{b_1}, \dots, \sigma_{b_p}$	Standard deviations of the regression coefficients
$d$	Number of independent variables in the regression equations plus one
$S_{YY}$	Total sum of squares about the mean
$S(\text{RES})$	Sum of squares about regression (residual)
$S(\text{REG})$	Sum of squares due to regression
$F$	Ratio for determining the statistical significance of a regression equation
$Y^c(U)$	Upper confidence limit for $Y^c$
$Y^c(L)$	Lower confidence limit for $Y^c$
CB-752	Columbium alloy base material made by Union Carbide
B-66	Columbium alloy base material made by Westinghouse

**APPLICATION OF REGRESSION ANALYSIS TECHNIQUES  
TO REFRACTORY COATING MEASUREMENT  
EXPERIMENTS**

**SUMMARY**

This report describes a procedure for conducting a statistical analysis of data obtained from two nondestructive techniques/instruments used to measure the thickness of protective refractory coatings on Columbium alloy. A regression analysis is performed on instrument output data to determine the significance of linear, quadratic, and cubic models. It is concluded that the linear regressions are the best choice of models for the particular coating, instrument, and base metals used.

**INTRODUCTION**

This report is the result of a request for computational support and analysis from the Materials Analysis Section of the Quality and Reliability Assurance Laboratory, Marshall Space Flight Center. A procedure is presented for conducting a statistical analysis of data obtained from two nondestructive techniques/instruments used to measure the thickness of protective refractory coatings on Columbium alloy. The two instruments used to obtain the test data are the Dermatron and the Betascope (a SPACO internal note gives a complete discussion of the test program and its relation to the Space Shuttle program<sup>1</sup>). The evaluation and analysis of data obtained from these instruments are required to evaluate accepted and reliable thickness measuring principles. Implementation of the various statistical methods required in the evaluation and analysis effort is accomplished on the UNIVAC 1108 computer. A complete program listing and input data setup examples are given in the appendix.

Regression analyses were performed on the output data from both instruments. Linear, quadratic, and cubic models were assumed and specific hypotheses tested concerning the significance of the various models. Confidence limits were attached to the models to establish a range of accuracy at a given confidence level. It was found that the linear regressions are the best choice of models for the particular coating, instrument, and base metals used. The accepted models allow instrument response to be related to actual coating thickness.

- 
1. Charles Wages and Marshall Parks: Evaluation of an Eddy Current and a Beta Backscatter Instrument for Measuring the Thickness of Refractory Coatings. Internal note, SPACO, Inc., Huntsville, Ala., Jan. 14, 1971.

# MATHEMATICAL DEVELOPMENT FOR MULTIPLE REGRESSION ANALYSIS

## General

We assume that the observed response variable (dependent variable)  $Y_i^O$  is to be estimated by the model:

$$Y_i^C = b_0 + b_1 Z_{1i} + b_2 Z_{2i} + \dots + b_p Z_{pi} \quad (1)$$

where  $i = 1, 2, \dots, n$ . If the input data are centered about the mean, this model becomes:

$$Y_i^C = b'_0 + b_1 z_{1i} + b_2 z_{2i} + \dots + b_p z_{pi} \quad (2)$$

where

$$\left. \begin{aligned} z_{1i} &= Z_{1i} - \bar{Z}_1 \\ z_{2i} &= Z_{2i} - \bar{Z}_2 \\ &\vdots \\ z_{pi} &= Z_{pi} - \bar{Z}_p \\ b'_0 &= b_0 + b_1 \bar{Z}_1 + b_2 \bar{Z}_2 + \dots + b_p \bar{Z}_p \end{aligned} \right\} \quad (3)$$

The sample estimates  $b'_0, b_1, b_2, \dots, b_p$  are obtained by minimizing the weighted sum of squares of deviations between the observed and predicted response values [1]. It can be verified from the first normal equation in the least-squares formulation that  $b'_0 = \bar{Y}$ . This leads to the following equation:

$$Y_i^C = \bar{Y} + b_1 z_{1i} + b_2 z_{2i} + \dots + b_p z_{pi} \quad (4)$$

Consideration of the remaining normal equations leads to the following equation for the regression coefficients:

$$\begin{matrix} \bar{B} \\ p \times 1 \end{matrix} = \begin{matrix} \bar{S}_p \\ p \times p \end{matrix} \begin{matrix} \bar{S}_Y \\ p \times 1 \end{matrix} \quad (5)$$

where

$$\begin{matrix} \bar{B} \\ p \times 1 \end{matrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_p \end{bmatrix} \quad (6)$$

$$\begin{matrix} \bar{S}_p \\ p \times p \end{matrix} = \begin{bmatrix} s_{11} & s_{12} & \dots & s_{1p} \\ s_{21} & s_{22} & \dots & s_{2p} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ s_{p1} & s_{p2} & \dots & s_{pp} \end{bmatrix}^{-1} \quad (7)$$

$$\begin{matrix} \bar{S}_Y \\ p \times 1 \end{matrix} = \begin{bmatrix} s_{1Y} \\ s_{2Y} \\ \vdots \\ \vdots \\ s_{pY} \end{bmatrix} \quad (8)$$

and the S's are given by:

$$\left. \begin{aligned} S_{IJ} &= \sum_{i=1}^n (Z_{Ii} - \bar{Z}_I) (Z_{Ji} - \bar{Z}_J) & I = 1, 2, \dots, p \text{ for} \\ & & \text{each } J = 1, 2, \dots, p \\ S_{IY} &= \sum_{i=1}^n (Z_{Ii} - \bar{Z}_I) (Y_i^O - \bar{Y}) & I = 1, 2, \dots, p \end{aligned} \right\} \quad (9)$$

The standard deviation of the dependent variable is given by:

$$\sigma_Y = \left[ \frac{\sum_{i=1}^n (Y_i^O - Y_i^C)^2}{n - d} \right]^{1/2} \quad (10)$$

where  $d = p + 1$ . For the regression coefficients we have:

$$\begin{matrix} \bar{\sigma}_R \\ (p+1) \times 1 \end{matrix} = \sigma_Y \begin{matrix} \bar{C} \\ (p+1) \times 1 \end{matrix} \quad (11)$$

where

$$\begin{matrix} \bar{\sigma}_R \\ (p+1) \times 1 \end{matrix} = \begin{bmatrix} \sigma_{\bar{Y}} \\ \sigma_{b_1} \\ \sigma_{b_2} \\ \cdot \\ \cdot \\ \sigma_{b_p} \end{bmatrix} \quad (12)$$

$$\begin{matrix} \bar{C} \\ (p+1) \times 1 \end{matrix} = \begin{bmatrix} 1/\sqrt{n} \\ \sqrt{C_{11}} \\ \sqrt{C_{22}} \\ \vdots \\ \sqrt{C_{pp}} \end{bmatrix} \quad (13)$$

The  $C$ 's are elements in the inverse of the  $S$  matrix as given by equation (7).

### Standard Error of $Y^c$

Consider equation (4). If  $\bar{Y}, b_1, b_2, \dots, b_p$  are subject to error and there is no correlation between  $\bar{Y}, b_1, b_2, \dots, b_p$ , the following error formula for the variance of  $Y^c$  can be written:

$$\sigma_{Y^c}^2 = \sigma_{\bar{Y}}^2 \left( \frac{\partial Y^c}{\partial \bar{Y}} \right)^2 + \sigma_{b_1}^2 \left( \frac{\partial Y^c}{\partial b_1} \right)^2 + \dots + \sigma_{b_p}^2 \left( \frac{\partial Y^c}{\partial b_p} \right)^2 \quad (14)$$

### Confidence Bands

The 95-percent confidence bands for the regression curve can now be established [2] by use of the Students  $T$  table. Let  $t_T$  denote the appropriate table value for  $N$  degrees of freedom where:

$$N = n - (p + 1) \quad (15)$$

Then the upper and lower bands for  $Y_i^c$  are determined from:

$$\left. \begin{aligned} Y_i^c(U) &= Y_i^c + t_T \sigma_{Y^c} \\ Y_i^c(L) &= Y_i^c - t_T \sigma_{Y^c} \end{aligned} \right\} \quad (16)$$

The resulting curves about the regression curve are the loci of the 95-percent confidence bands. These bands can be interpreted as follows. If repeated observations of  $Y_k^O$  are taken and at the same fixed values of  $Z_{1k}, Z_{2k}, \dots, Z_{pk}$  as were used to determine the fitted regression equation, then of all the 95-percent confidence intervals constructed for the mean value of  $Y_k^O$ , 95 percent of these intervals will contain the true mean value of  $Y_k^O$ ; or, there is a 0.95 probability that the following statement is correct:

“The true mean value of  $Y_k^O$  at  $Z_{1k}, Z_{2k}, \dots, Z_{pk}$  lies in the interval  $[Y_k^C \pm t_T \sigma_{Y^C}]$ .”

### Significance of the Estimated Regression Equation

We consider the following analysis of variance table for the regression equation as given by equation (4).

TABLE 1. ANALYSIS OF VARIANCE

df	Type Variation	SS	MS	F Value
n - 1	Total	$S_{YY} = \sum_{i=1}^n (Y_i^O - \bar{Y})^2$		
n - d	Residual	$S(\text{RES}) = \sum_{i=1}^n (Y_i^O - Y_i^C)^2$	$M(\text{RES}) = \frac{S(\text{RES})}{n - d}$	
d - 1	Regression	$S(\text{REG}) = \sum_{i=1}^n (Y_i^C - \bar{Y})^2$	$M(\text{REG}) = \frac{S(\text{REG})}{d - 1}$	$\frac{M(\text{REG})}{M(\text{RES})}$
df = degrees of freedom SS = sum of squares MS = mean square = SS/df n = number of observations p = number of independent variables d = p + 1				

The total sum of squares can be written as:

$$S_{YY} = S(\text{RES}) + S(\text{REG}) \quad (17)$$

If  $S(\text{RES}) = 0$ , the actual observations of the dependent variable are described exactly by the regression equation (4). The ratio defined by

$$F = \frac{S(\text{REG})/(d - 1)}{S(\text{RES})/(n - d)} \quad (18)$$

follows an F distribution with  $(d - 1)$  and  $(n - d)$  degrees of freedom. This quantity is used to determine the statistical significance of the regression equation under consideration by comparing it with the appropriate F table value. If the computed F value [equation (18)] is greater than the appropriate F table value, the regression equation is statistically significant.

### Application

The application of the preceding analysis to film thickness measurement data is concerned with the following equation:

$$Y_i^c = b_0 + b_1 X_{1i} + b_2 X_{1i}^2 + b_3 X_{1i}^3 \quad (19)$$

where we wish to determine whether we should use  $X_{1i}^2$  and/or  $X_{1i}^3$  in the equation. This equation can be put in the form:

$$Y_i^c = b'_0 + b_1 Z_{1i} + b_2 Z_{2i} + b_3 Z_{3i} \quad (20)$$

where

$$\left. \begin{aligned} Z_{1i} &= X_{1i} - \bar{X}_1 \\ Z_{2i} &= X_{1i}^2 - \bar{X}_1^2 \\ Z_{3i} &= X_{1i}^3 - \bar{X}_1^3 \\ b'_0 &= \bar{Y} \end{aligned} \right\} \quad (21)$$

with

$$\left. \begin{aligned} \bar{X}_1 &= \left( \sum_{i=1}^n X_{1i} \right) / n \\ \bar{X}_1^2 &= \left( \sum_{i=1}^n X_{1i}^2 \right) / n \\ \bar{X}_1^3 &= \left( \sum_{i=1}^n X_{1i}^3 \right) / n \\ b_0 &= b'_0 - b_1 \bar{X}_1 - b_2 \bar{X}_1^2 - b_3 \bar{X}_1^3 \end{aligned} \right\} \quad (22)$$

The solution for the regression coefficients in equation (20) (Model 3) then becomes:

$$\begin{matrix} \bar{B} \\ 3 \times 1 \end{matrix} = \begin{matrix} \bar{S}_3 \\ 3 \times 3 \end{matrix} \begin{matrix} \bar{S}_Y \\ 3 \times 1 \end{matrix}, \quad (23)$$

where

$$\begin{matrix} \bar{B} \\ 3 \times 1 \end{matrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad (24)$$

$$\begin{matrix} \bar{S}_3 \\ 3 \times 3 \end{matrix} = \begin{bmatrix} \sum Z_{1i} Z_{1i} & \sum Z_{1i} Z_{2i} & \sum Z_{1i} Z_{3i} \\ \sum Z_{2i} Z_{1i} & \sum Z_{2i} Z_{2i} & \sum Z_{2i} Z_{3i} \\ \sum Z_{3i} Z_{1i} & \sum Z_{3i} Z_{2i} & \sum Z_{3i} Z_{3i} \end{bmatrix}^{-1} \quad (25)$$

$$\bar{S}_Y = \begin{bmatrix} \sum Z_{1i} Y_i^o \\ \sum Z_{2i} Y_i^o \\ \sum Z_{3i} Y_i^o \end{bmatrix}_{3 \times 1} \quad (26)$$

Thus, we can determine the regression coefficients for the following models:

$$\text{Model 1: } Y_i^c = b'_0 + b_1 Z_{1i}$$

$$\text{Model 2: } Y_i^c = b'_0 + b_1 Z_{1i} + b_2 Z_{2i} \quad ,$$

and

$$\text{Model 3: } Y_i^c = b'_0 + b_1 Z_{1i} + b_2 Z_{2i} + b_3 Z_{3i}$$

These three models yield the following sum of squares due to regression:

$$\text{Model 1: } S(\text{REG})_1$$

$$\text{Model 2: } S(\text{REG})_2$$

and

$$\text{Model 3: } S(\text{REG})_3$$

We now want to test the following hypothesis:

$$H_1 : X_1^2 \text{ contributes nothing to variation in } Y (\text{i.e., } b_2 = 0) .$$

$$H_2 : X_1^3 \text{ contributes nothing to variation in } Y (\text{i.e., } b_3 = 0) .$$

The test criteria of  $H_1$  and  $H_2$  are:

$$\left. \begin{aligned} F_{c1} &= \frac{S_{2,1}}{S_{YY}} \quad , \text{ with 1 and } n - 3 \text{ df} \\ F_{c2} &= \frac{S_{3,2}}{S_{YY}} \quad , \text{ with 1 and } n - 4 \text{ df} \end{aligned} \right\} \quad (27)$$

where

$$\left. \begin{aligned} S_{2,1} &= S(\text{REG})_2 - S(\text{REG})_1 \\ S_{3,2} &= S(\text{REG})_3 - S(\text{REG})_2 \end{aligned} \right\} \quad (28)$$

We determine the 95-percent points  $F[1, n-3]$  and  $F[1, n-4]$  in the  $F$  table. If  $F_{c1} < F_T$ , we accept  $H_1$ . Otherwise, reject  $H_1$  and say that  $X_1^2$  adds significantly to the linear regression. If  $F_{c2} < F_T$ , we accept  $H_2$ . If  $F_{c2} \geq F_T$ , then  $H_2$  is rejected, and we can conclude that  $X_1^3$  adds significantly to the linear regression.

## RESULTS AND CONCLUSIONS

The basic input data for the analysis consist of specimen thickness measurements (independent variable) and instrument meter readings (dependent variable). The thickness measurements are in mils and the meter readings in counts for the Betascope and voltages for the Dermitron. Table 2 is a summary of the six samples from which the basic input data were obtained. As seen from this table, the data for a given sample correspond to a particular type of coating, base metal, and instrument. A complete discussion of these data and the test program from which they were obtained is given in a SPACO internal note<sup>2</sup>.

Linear, quadratic, and cubic regression analyses were performed on the data for each sample. As shown in Table 3, the linear models for the six samples were statistically significant based on the  $F$  test. The hypotheses  $H_1$  and  $H_2$  discussed in the section Mathematical Development for Multiple Regression Analysis, concerning the quadratic and cubic regression models were then tested. These results are summarized in Table 4. It is indicated in this table that for the particular coating, instrument, and base metal used in Table 2, the squared and cubic terms postulated in the models did not contribute significantly to the regression. It was, therefore, concluded that the linear regressions in Figures 1 through 6 are the best choices of models for the indicated data. Confidence limits given in these figures establish a range of accuracy at the 95-percent confidence level.

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2. Ibid.

TABLE 2. SUMMARY OF INPUT DATA SOURCE  
FOR THE REGRESSION ANALYSIS

Sample No.	Instrument	Coating	Base Metal	Number of Measurements
1	Dermatron	LB-2	CB-752	23
2	Dermatron	SYLCOR	CB-752	33
3	Dermatron	SYLCOR	B-66	26
4	Betascope	LB-2	CB-752	20
5	Betascope	SYLCOR	CB-752	33
6	Betascope	SYLCOR	B-66	26

TABLE 3. F VALUES FOR THE LINEAR REGRESSION MODELS

Sample No.	n	$F_T [1, n - 2], 0.95$ Table Value	$F_C$ , Computed Value
1	23	4.28	608.4
2	33	4.14	729.9
3	26	4.23	455.3
4	20	4.35	87.2
5	33	4.14	502.4
6	26	4.23	161.2

TABLE 4. F VALUES FOR TESTING HYPOTHESES  $H_1$  AND  $H_2^a$

Sample No.	n	F <sub>T</sub> , Table Value		F <sub>c</sub> , Computed	
		F <sub>T</sub> [1, n - 3]	F <sub>T</sub> [1, n - 4]	F <sub>c1</sub>	F <sub>c2</sub>
1	23	4.35	4.38	0.02	0.0001
2	33	4.17	4.18	0.001	0.007
3	26	4.28	4.30	0.02	0.0003
4	20	4.45	4.49	0.01	0.007
5	33	4.17	4.18	0.03	0.00005
6	26	4.28	4.30	0.05	0.002

a. Hypothesis

$H_1 : X_1^2$  contributes nothing to variation in Y.

$H_2 : X_1^3$  contributes nothing to variation in Y.

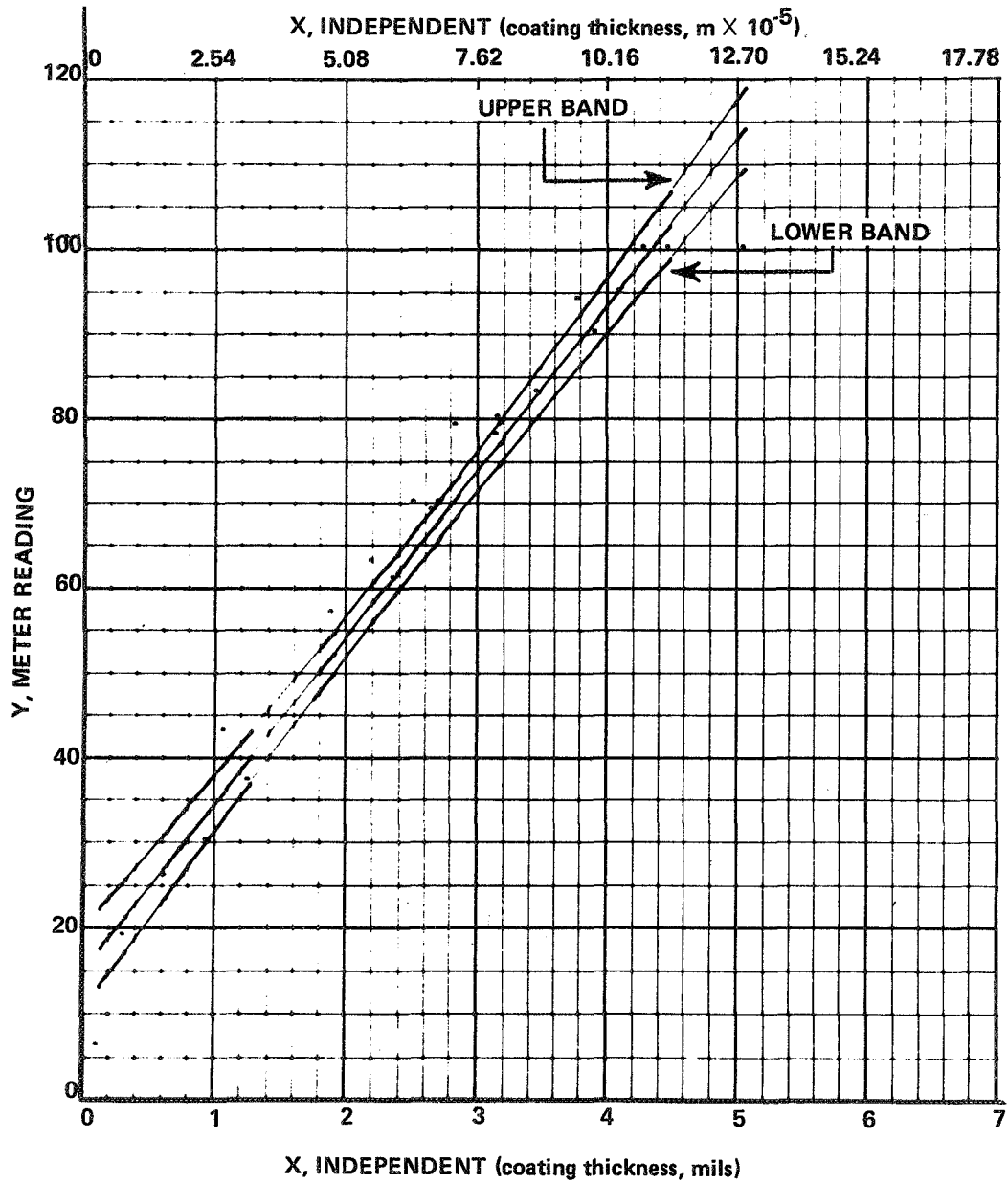


Figure 1. Linear regression for Dermitron measurements on sample No. 1.

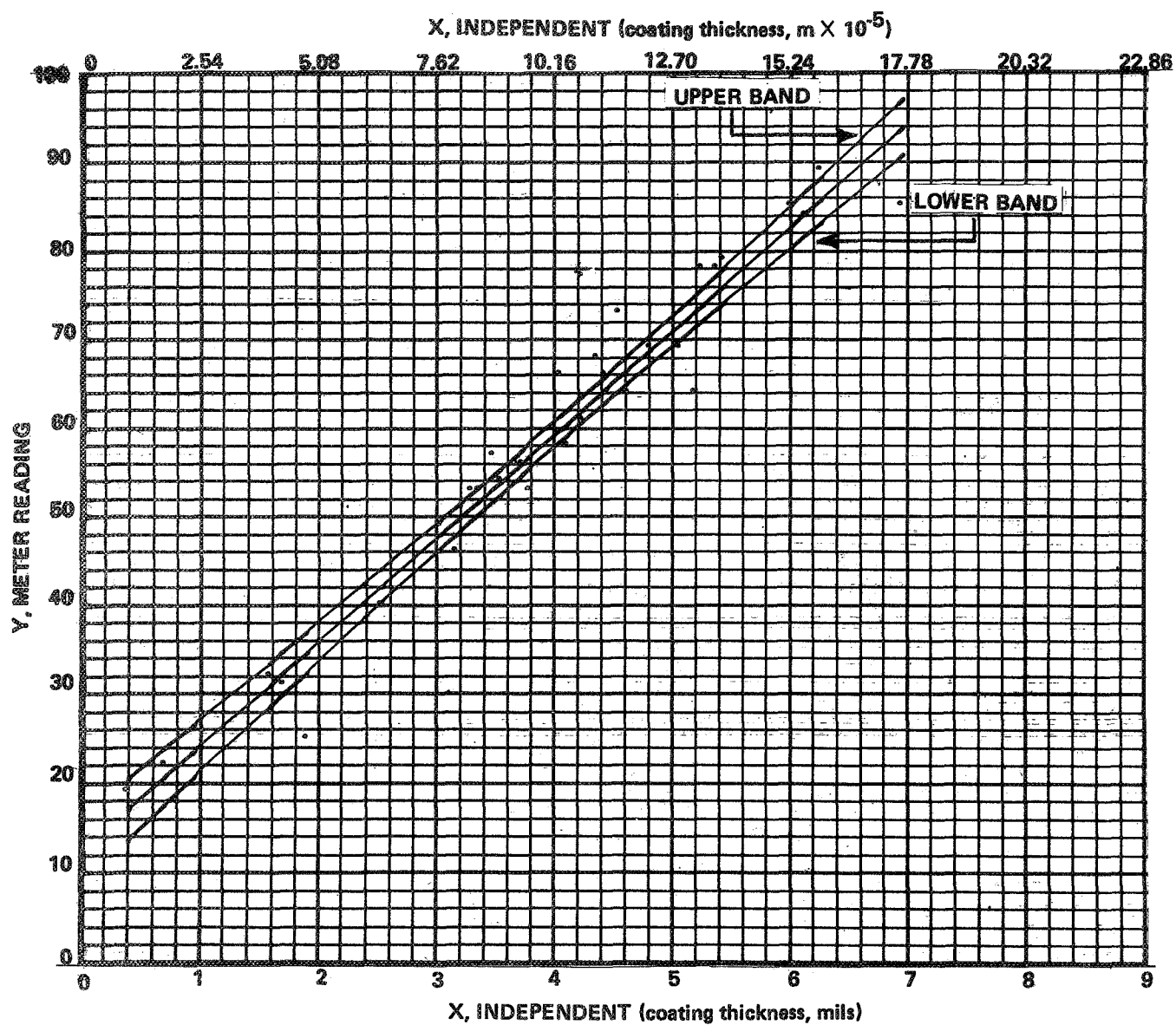


Figure 2. Linear regression for Dermatron measurements on sample No. 2.

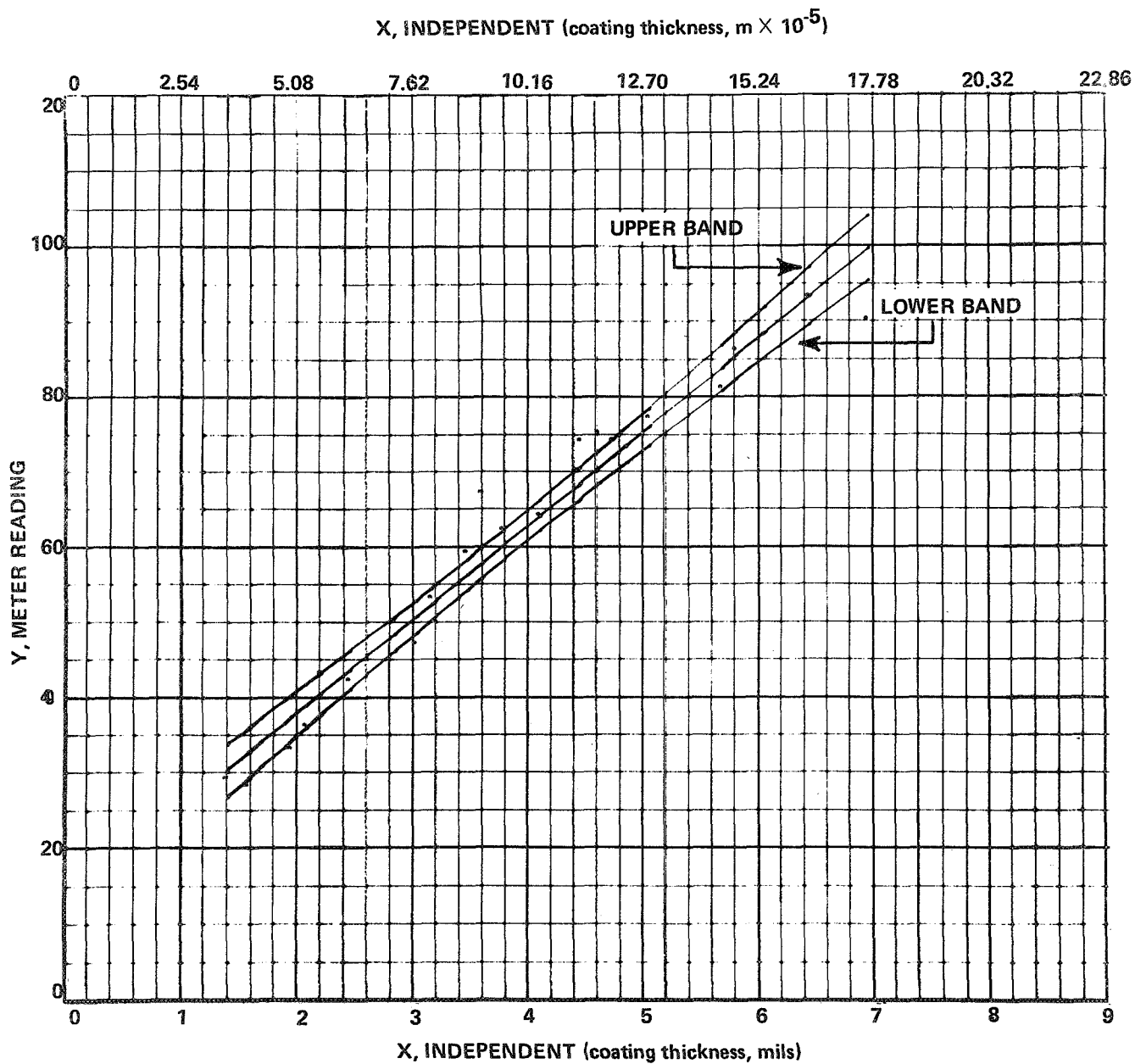


Figure 3. Linear regression for Dermatron measurements on sample No. 3.

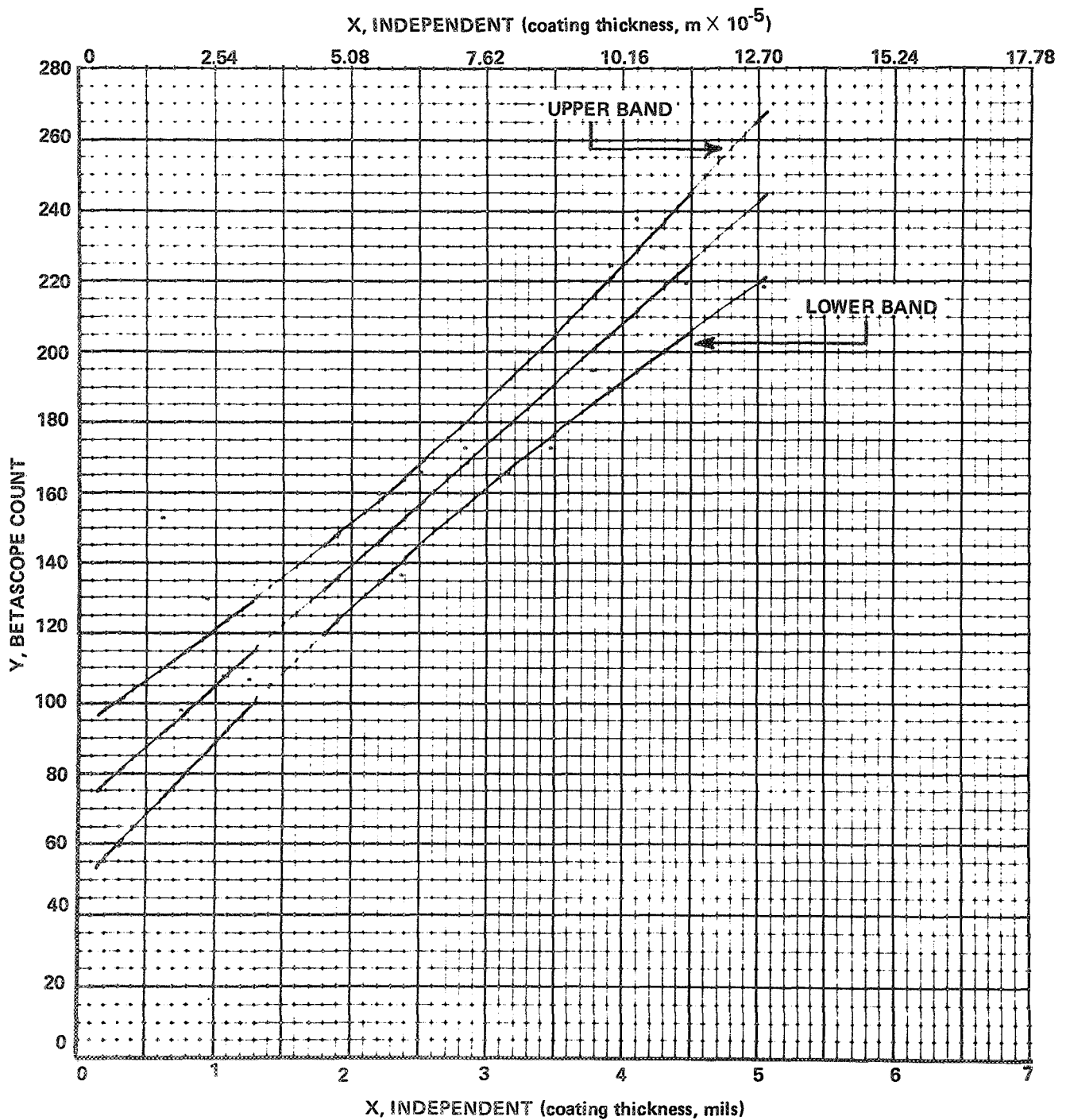


Figure 4. Linear regression for Betascope measurements on sample No. 4.

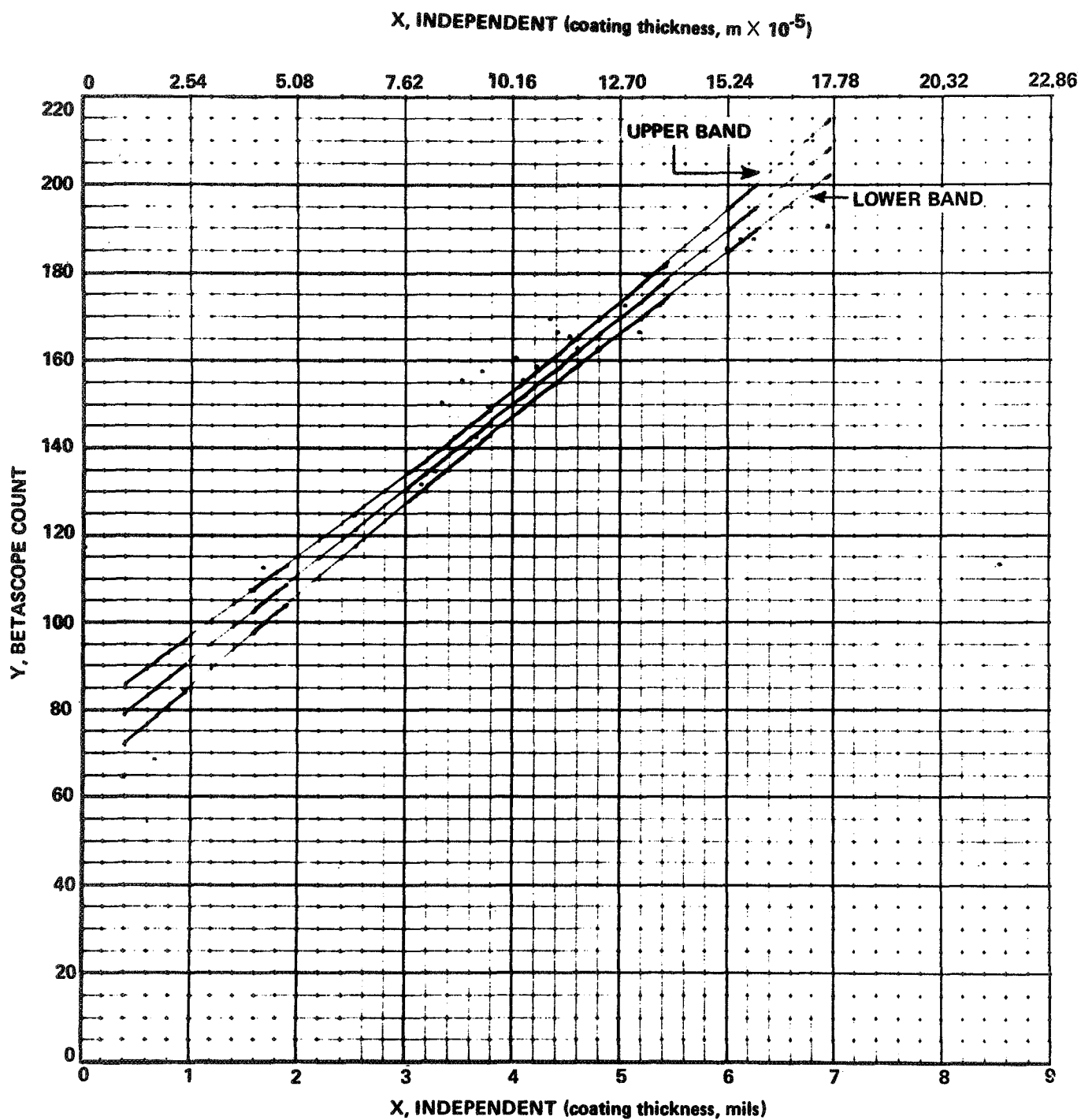


Figure 5. Linear regression for Betascope measurements on sample No. 5.

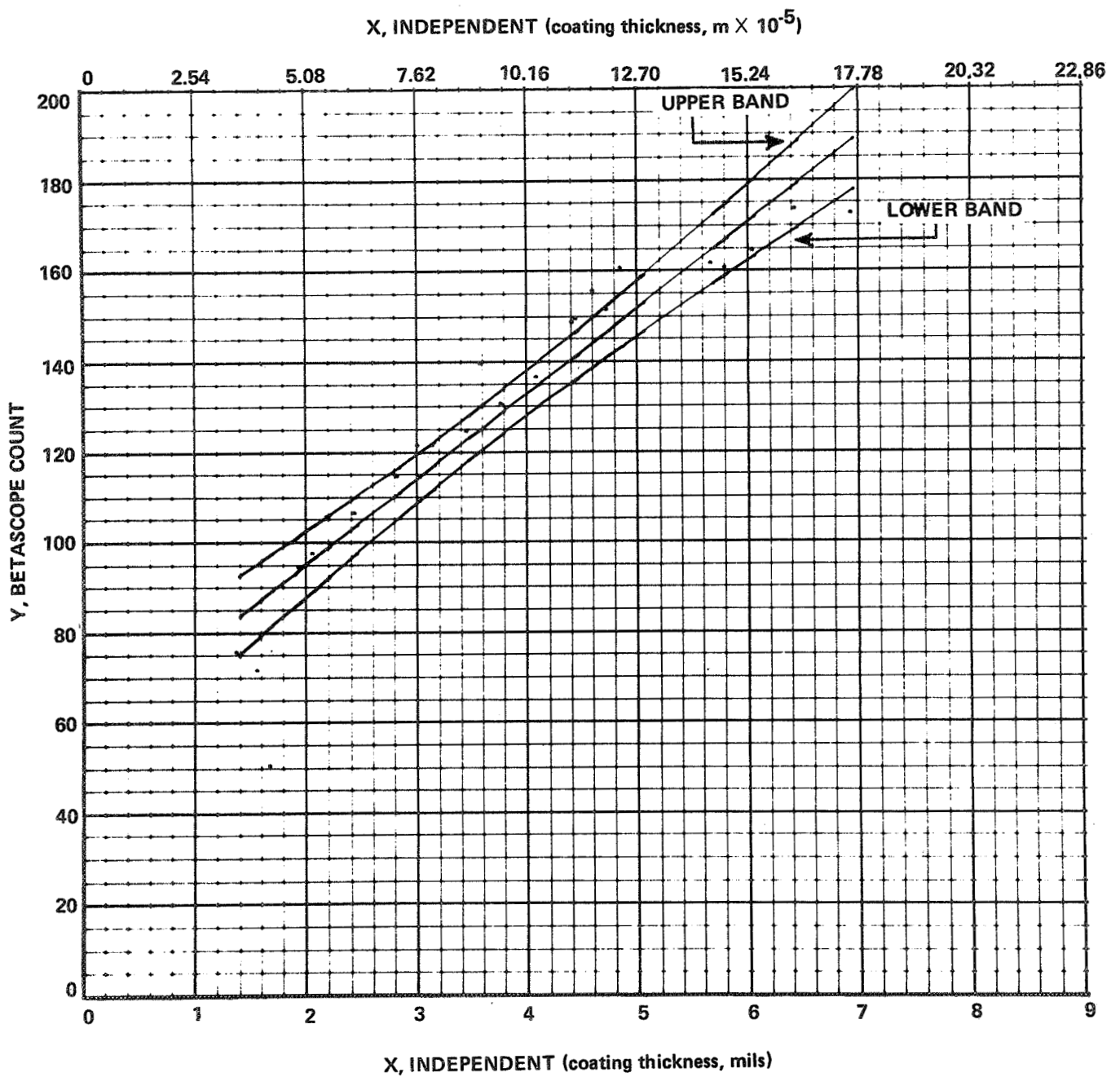


Figure 6. Linear regression for Betascope measurements on sample No. 6.

## APPENDIX

### COMPUTER PROGRAM DOCUMENTATION

This appendix presents operational information on the UNIVAC 1108 Computer Program for the regression analysis application to film thickness measurements experiments. The organization of the operational version of the program is depicted in Figure A-1. As shown, a minimum number of control cards are required to run this deck. A complete listing of the program for a typical run is included.

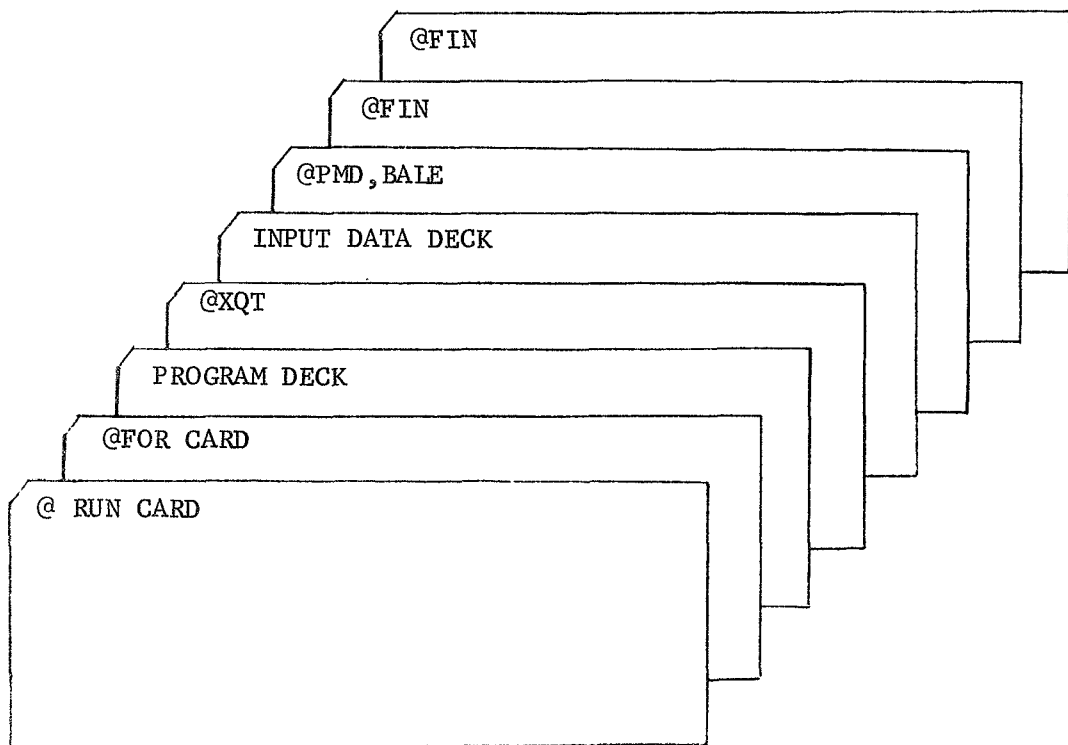


Figure A-1. Program organization.

## PROGRAM LISTING

```

-RUN,T REGRES,310390,JUNKINBIN313,1,100
-FOR,IS LQCREG,LQCREG
C
C  REGRESSION ANALYSIS APPLICATION TO FILM THICKNESS
C  MEASUREMENT EXPERIMENTS
C
      DIMENSION X(100),Y(100),YC1(100),RES1(100),REG1(100),TOTAL(100),YC
-2(100),RES2(100),REG2(100),YC3(100),RES3(100),REG3(100),D1(20,2),D
-2(20,2),D3(20,3),YB1(100),YB2(100),YB3(100),YB4(100),YB5(100),YB6(
-100),BCDX(12),BCDY(12)
      DIMENSION DD2(2,2),DD3(3,3),BJ2(2,2),BJ3(3,3)
      DIMENSION Z1(100),Z2(100),Z3(100),YH(100)
      DIMENSION VARY1(100),VARY2(100),VARY3(100),SGY1(100),SGY2(100),
-SGY3(100)
      DATA (BCDX(I),I=1,12)/6HX,INDE,6HPENDEN,6HT      ,9*6H      /
      DATA (BCDY(I),I=1,12)/6HY      ,11*6H      /
100  FORMAT (1H1)
101  FORMAT (//)
102  FORMAT (1H1,74H REGRESSION ANALYSIS APPLICATION TO FILM THICKNESS
- MEASUREMENT EXPERIMENTS)
103  FORMAT (64H LINEAR,QUADRATIC,AND CUBIC REGRESSION ON INPUT MEASURE
- MENT DATA)
104  FORMAT (2X,13HX,INDEPENDENT,3X,11HY,DEPENDENT,3X,12HYC1,DEGREE-1,3
- X,12HYC2,DEGREE-2,3X,12HYC3,DEGREE-3)
105  FORMAT (2X,F6.2,10X,F8.3,6X,F8.3,7X,F8.3,7X,F8.3)
106  FORMAT (31H LINEAR REGRESSION COEFFICIENTS)
107  FORMAT (3X,3HRO=E13.6,3X,3HR1=E13.6,3X,8HF VALUE=E13.6)
108  FORMAT (3X,8HSIGMA Y=E13.6,3X,5HSGRO=E13.6,3X,5HSGR1=E13.6)
109  FORMAT (34H QUADRATIC REGRESSION COEFFICIENTS)
110  FORMAT (3X,3HSO=E13.6,3X,3HS1=E13.6,3X,3HS2=E13.6,3X,8HF VALUE=E13
- .6)
111  FORMAT (3X,8HSIGMA Y=E13.6,3X,5HSGSO=E13.6,3X,5HSGS1=E13.6,3X,5HSG
- S2=E13.6)
112  FORMAT (42H F VALUE FOR SIGNIFICANCE OF X-SQUARE TERM)
113  FORMAT (3X,4HF21=E13.6)
114  FORMAT (30H CUBIC REGRESSION COEFFICIENTS)
115  FORMAT (3X,3HTO=E13.6,3X,3HT1=E13.6,3X,3HT2=E13.6,3X,3HT3=E13.6,3X
- ,8HF VALUE=E13.6)
116  FORMAT (3X,8HSIGMA Y=E13.6,3X,5HSGTO=E13.6,3X,5HSGT1=E13.6,3X,5HSG
- T2=E13.6,3X,5HSGT3=E13.6)
117  FORMAT (40H F VALUE FOR SIGNIFICANCE OF X-CUBE TERM)
118  FORMAT (3X,4HF32=E13.6)
119  FORMAT (54H DETERMINANT VALUE FOR QUADRATIC AND CUBIC REGRESSIONS)
120  FORMAT (3X,5HDET2=E13.6,3X,5HDET3=E13.6)
121  FORMAT (33H INVERSE MATRIX X ORIGINAL MATRIX)
123  FORMAT (21H QUADRATIC REGRESSION)
124  FORMAT (17H CUBIC REGRESSION)
125  FORMAT (3X,9(E13.6,1X))
126  FORMAT (13)
      D1(1,1)=0.

```

# PROGRAM LISTING (Continued)

```

      NAMELIST/INPUT/X,Y,TVAL1,TVAL2,TVAL3,NN,XL,XR,YB,YT
      CALL IDENT(935)
      INTEGER P
      READ (5,126) NCASES
300   WRITE (6,100)
      READ (5,INPUT)
      WRITE (6,INPUT)
C
C   INITIALIZATION SUMMATIONS
C
      BJ=NN
      AX1=0.
      AX2=0.
      AX3=0.
      AY1=0.
      DO 301 I=1,NN
      AX1=X(I)+AX1
      AX2=X(I)*X(I)+AX2
      AX3=X(I)**3+AX3
301   AY1=Y(I)+AY1
      BX1=AX1/BJ
      BX2=AX2/BJ
      BX3=AX3/BJ
      BY=AY1/BJ
      AVY=BY
      DO 3010 I=1,NN
      Z1(I)=X(I)-BX1
      Z2(I)=X(I)*X(I)-BX2
      Z3(I)=X(I)**3-BX3
3010  YH(I)=Y(I)-BY
      S11=0.
      S12=0.
      S13=0.
      S21=0.
      S22=0.
      S23=0.
      S31=0.
      S32=0.
      S33=0.
      S1Y=0.
      S2Y=0.
      S3Y=0.
      DO 3011 I=1,NN
      S11=Z1(I)*Z1(I)+S11
      S12=Z1(I)*Z2(I)+S12
      S13=Z1(I)*Z3(I)+S13
      S22=Z2(I)*Z2(I)+S22
      S23=Z2(I)*Z3(I)+S23
      S33=Z3(I)*Z3(I)+S33
      S1Y=Z1(I)*YH(I)+S1Y
      S2Y=Z2(I)*YH(I)+S2Y
3011  S3Y=Z3(I)*YH(I)+S3Y
      S21=S12
      S31=S13
      S32=S23

```

## PROGRAM LISTING (Continued)

```

C
C  LINEAR REGRESSION ANALYSIS
C
      R1=S1Y/S11
      R0=BY-R1*B1
      DO 302 I=1,NN
C  COMPUTED DEPENDENT VARIABLE (CDV)
      YC1(I)=R0+R1*X(I)
C  RESIDUAL VARIATIONS (RFSV)
      RFS1(I)=(Y(I)-YC1(I))**2
C  REGRESSION VARIATIONS (REGV)
      RFG1(I)=(YC1(I)-AVY)**2
C  TOTAL VARIATION (TOTV)
302  TOTAL(I)=(Y(I)-AVY)**2
      SRFS1=0.
      SREG1=0.
      SYY=0.
      DO 303 I=1,NN
C  RESIDUAL SUM OF SQS. (RESSQ)
      SRFS1=RFS1(I)+SRFS1
C  REGRESSION SUM OF SQS. (REGSQ)
      SREG1=RFG1(I)+SREG1
C  TOTAL SUM OF SQS. (TOTSQ)
303  SYY=TOTAL(I)+SYY
      RM1=SRFS1/(NJ-2.)
      RM2=SREG1
C  F VALUE
      FVALU1=RM2/RM1
C
C  QUADRATIC REGRESSION ANALYSIS
C
      D2(1,1)=S11
      D2(1,2)=S12
      D2(2,1)=S12
      D2(2,2)=S22
      DO 3030 I=1,2
      DO 3030 J=1,2
3030  DD2(I,J)=D2(I,J)
      CALL GASINV(D2,2,DET2)
      DO 3031 I=1,2
      DO 3031 J=1,2
3031  BJ2(I,J)=0.
      DO 3032 M=1,2
      DO 3032 P=1,2
      DO 3032 N=1,2
3032  BJ2(M,P)=D2(M,V1)*DD2(N,P)+BJ2(M,P)
      S1=D2(1,1)*S1Y+D2(1,2)*S2Y
      S2=D2(2,1)*S1Y+D2(2,2)*S2Y
      S0=BY-S1*B1-S2*B2
      DO 304 I=1,NN
C      CDV
      YC2(I)=S0+S1*X(I)+S2*X(I)*X(I)
C      RFSV
      RES2(I)=(Y(I)-YC2(I))**2
C      RFGV

```

# PROGRAM LISTING (Continued)

```

304  REG2(I)=(YC2(I)-AVY)**2
      SRFS2=0.
      SREG2=0.
      DO 305 I=1,NN
C      RESSQ
      SRES2=RES2(I)+SRES2
C      REGSQ
305  SREG2=REG2(I)+SREG2
      SM1=SRES2/(BJ-3.)
      SM2=SREG2/2.
C  F VALUE
      FVALU2=SM2/SM1
C
C  CUBIC REGRESSION ANALYSIS
C
      D3(1,1)=S11
      D3(1,2)=S12
      D3(1,3)=S13
      D3(2,1)=S12
      D3(2,2)=S22
      D3(2,3)=S23
      D3(3,1)=S13
      D3(3,2)=S23
      D3(3,3)=S33
      DO 3050 I=1,3
      DO 3050 J=1,3
3050 DD3(I,J)=D3(I,J)
      CALL GASINV(D3,3,DET3)
      DO 3051 I=1,3
      DO 3051 J=1,3
3051 BJ3(I,J)=0.
      DO 3052 M=1,3
      DO 3052 P=1,3
      DO 3052 N=1,3
3052 BJ3(M,P)=D3(M,N)*DD3(N,P)+BJ3(M,P)
      T1=D3(1,1)*S1Y+D3(1,2)*S2Y+D3(1,3)*S3Y
      T2=D3(2,1)*S1Y+D3(2,2)*S2Y+D3(2,3)*S3Y
      T3=D3(3,1)*S1Y+D3(3,2)*S2Y+D3(3,3)*S3Y
      T0=BY-T1*BX1-T2*BX2-T3*BX3
      DO 306 I=1,NN
C      CDV
      YC3(I)=T0+T1*X(I)+T2*X(I)*X(I)+T3*(X(I)**3)
C      RESV
      RES3(I)=(Y(I)-YC3(I))**2
C      REGV
306  REG3(I)=(YC3(I)-AVY)**2
      SRFS3=0.
      SREG3=0.
      DO 307 I=1,NN
C      RESSQ
      SRES3=RES3(I)+SRES3
C      REGSQ
307  SREG3=REG3(I)+SREG3
      TM1=SRES3/(BJ-4.)
      TM2=SREG3/3.

```

## PROGRAM LISTING (Continued)

```

C  F VALUE
    FVALU3=TM2/TM1
C
C  TEST FOR SIGNIFICANCE OF INDIVIDUAL TERMS
C
    W21=SREG2-SREG1
    W32=SREG3-SREG2
    F21=W21/SYY
    F32=W32/SYY
C
C  STANDARD DEVIATION OF DEPENDENT VARIABLE FOR LINEAR,
C  QUADRATIC, AND CUBIC REGRESSION
C
    SIGMA1=SQRT(SRES1/(RJ-2.))
    SIGMA2=SQRT(SRES2/(RJ-3.))
    SIGMA3=SQRT(SRES3/(RJ-4.))
C
C  STANDARD DEVIATIONS FOR LINEAR, QUADRATIC, AND
C  CUBIC REGRESSION COEFFICIENTS
C
    SGR0=SIGMA1/SQRT(RJ)
    SGR1=SIGMA1/SQRT(S11)
    SGS0=SIGMA2/SQRT(RJ)
    SGS1=SIGMA2*SQRT(D2(1,1))
    SGS2=SIGMA2*SQRT(D2(2,2))
    SGT0=SIGMA3/SQRT(RJ)
    SGT1=SIGMA3*SQRT(D3(1,1))
    SGT2=SIGMA3*SQRT(D3(2,2))
    SGT3=SIGMA3*SQRT(D3(3,3))
C
C  CONFIDENCE BANDS FOR LINEAR, QUADRATIC AND CUBIC REGRESSIONS
C
    DO 308 I=1,NN
    ZZ1=Z1(I)**2
    ZZ2=Z2(I)**2
    ZZ3=Z3(I)**2
    VARY1(I)=SGR0*SGR0+SGR1*SGR1*ZZ1
    VARY2(I)=SGS0*SGS0+SGS1*SGS1*ZZ1+SGS2*SGS2*ZZ2
    VARY3(I)=SGT0*SGT0+SGT1*SGT1*ZZ1+SGT2*SGT2*ZZ2+SGT3*SGT3*ZZ3
    SGY1(I)=SQRT(VARY1(I))
    SGY2(I)=SQRT(VARY2(I))
    SGY3(I)=SQRT(VARY3(I))
    YB1(I)=YC1(I)+TVAL1*SGY1(I)
    YB2(I)=YC1(I)-TVAL1*SGY1(I)
    YB3(I)=YC2(I)+TVAL2*SGY2(I)
    YB4(I)=YC2(I)-TVAL2*SGY2(I)
    YB5(I)=YC3(I)+TVAL3*SGY3(I)
308  YB6(I)=YC3(I)-TVAL3*SGY3(I)
    IPAGE=48
    WRITE (6,101)
    WRITE (6,102)
    WRITE (6,101)
    WRITE (6,103)
    WRITE (6,101)
    WRITE (6,104)

```

# PROGRAM LISTING (Continued)

```

DO 311 I=1,NN
WRITE (6,105) X(I),Y(I),YC1(I),YC2(I),YC3(I)
IPAGE=IPAGE-1
IF (IPAGE) 310,309,310
309 WRITE (6,100)
WRITE (6,101)
WRITE (6,104)
310 CONTINUE
311 CONTINUE
WRITE (6,100)
WRITE (6,106)
WRITE (6,107) R0,R1,FVALU1
WRITE (6,108) SIGMA1,SGR0,SGR1
WRITE (6,101)
WRITE (6,109)
WRITE (6,110) S0,S1,S2,FVALU2
WRITE (6,111) SIGMA2,SGS0,SGS1,SGS2
WRITE (6,112)
WRITE (6,113) F21
WRITE (6,101)
WRITE (6,114)
WRITE (6,115) T0,T1,T2,T3,FVALU3
WRITE (6,116) SIGMA3,SGT0,SGT1,SGT2,SGT3
WRITE (6,117)
WRITE (6,118) F32
WRITE (6,119)
WRITE (6,120) DET2,DET3
IW=2
I=1
WRITE (6,101)
WRITE (6,121)
WRITE (6,123)
400 WRITE (6,125) (BJ2(I,J),J=1,2)
IW=IW-1
IF (IW.EQ.0) GO TO 401
I=I+1
GO TO 400
401 CONTINUE
IW=3
I=1
WRITE (6,101)
WRITE (6,124)
402 WRITE (6,125) (BJ3(I,J),J=1,3)
IW=IW-1
IF (IW.EQ.0) GO TO 403
I=I+1
GO TO 402
403 CONTINUE
KK=NN
CALL QUIK3L(-1,XL,XR,YB,YT,43,BCDX,BCDY,KK,X,Y)
CALL QUIK3L(0,XL,XR,YB,YT,35,BCDX,BCDY,-KK,X,YB1)
CALL QUIK3L(0,XL,XR,YB,YT,35,BCDX,BCDY,-KK,X,YC1)
CALL QUIK3L(0,XL,XR,YB,YT,35,BCDX,BCDY,-KK,X,YB2)
CALL QUIK3L(-1,XL,XR,YB,YT,43,BCDX,BCDY,KK,X,Y)
CALL QUIK3L(0,XL,XR,YB,YT,35,BCDX,BCDY,-KK,X,YB3)

```

# PROGRAM LISTING (Continued)

```

CALL QUIK3L(0,XL,XR,YB,YT,35,BCDX,BCDY,-KK,X,YC2)
CALL QUIK3L(0,XL,XR,YB,YT,35,BCDX,BCDY,-KK,X,YB4)
CALL QUIK3L(-1,XL,XR,YB,YT,43,BCDX,BCDY,KK,X,Y)
CALL QUIK3L(0,XL,XR,YB,YT,35,BCDX,BCDY,-KK,X,YB5)
CALL QUIK3L(0,XL,XR,YB,YT,35,BCDX,BCDY,-KK,X,YC3)
CALL QUIK3L(0,XL,XR,YB,YT,35,BCDX,BCDY,-KK,X,YB6)
NCASES=NCASES-1
IF (NCASES.EQ.0) GO TO 312
GO TO 300
312 CONTINUE
CALL ENDJOB
STOP
END
-XQT
+06
$INPUT
X=.13,.32,.63,.76,.95,1.08,1.27,1.90,2.21,2.37,2.53,2.66,2.72,2.85,
3.16,3.17,3.48,3.79,3.92,4.11,4.30,4.48,5.06,
Y=6.,19.,26.,29.,30.,43.,37.,57.,63.,61.,70.,69.,70.,79.,78.,80.,83.,
94.,90.,95.,100.,100.,100., TVAL1=2.080, TVAL2=2.086, TVAL3=2.093, NN=23,
XL=0.,XR=10.,YB=0.,YT=150.,
$
$INPUT
X=.38,.70,.95,1.58,1.70,1.89,2.53,3.03,3.16,3.29,3.35,3.48,3.54,3.67,
3.73,3.79,4.05,4.11,4.24,4.36,4.43,4.55,4.62,4.81,5.06,5.19,5.25,5.38,
5.44,6.01,6.14,6.26,6.96,
Y=19.,22.,23.,32.,31.,25.,40.,49.,46.,53.,53.,57.,54.,56.,56.,53.,66.,
58.,61.,68.,66.,73.,64.,69.,69.,64.,78.,78.,79.,85.,84.,89.,85., TVAL1=2.040,
TVAL2=2.042, TVAL3=2.045, NN=33,
XL=0.,XR=10.,YB=0.,YT=130.,
$
$INPUT
X=1.39,1.58,1.70,1.89,1.96,2.08,2.21,2.46,2.84,3.03,3.16,3.48,3.60,
3.79,4.11,4.43,4.46,4.62,4.74,4.87,5.06,5.69,5.82,6.07,6.45,6.96,
Y=29.,28.,23.,46.,33.,36.,43.,42.,50.,47.,53.,59.,67.,62.,64.,70.,
74.,75.,74.,73.,77.,81.,86.,88.,93.,90., TVAL1=2.064, TVAL2=2.069, TVAL3=2.074,
NN=26,
XL=0.,XR=10.,YB=0.,YT=150.,
$
$INPUT
X=.13,.32,.63,.76,.95,1.08,1.27,1.90,2.21,2.38,2.53,2.85,3.17,3.48,
3.79,3.92,4.11,4.30,4.48,5.06,
Y=19.,59.,152.,97.,129.,107.,106.,147.,159.,136.,165.,172.,165.,172.,
194.,224.,237.,229.,219.,218., TVAL1=2.101, TVAL2=2.110, TVAL3=2.120, NN=20,
XL=0.,XR=8.,YB=0.,YT=300.,
$
$INPUT
X=.38,.69,.95,1.58,1.70,1.89,2.53,3.03,3.16,3.29,3.35,3.48,3.54,3.67,
3.73,3.79,4.05,4.11,4.24,4.36,4.43,4.55,4.62,4.81,5.06,5.19,5.25,
5.38,5.44,6.01,6.14,6.26,6.96,
Y=64.,68.,84.,107.,112.,103.,124.,130.,131.,134.,150.,142.,155.,142.,
157.,149.,160.,155.,158.,169.,166.,165.,162.,162.,172.,166.,179.,172.,
181.,185.,187.,187.,190., TVAL1=2.040, TVAL2=2.042, TVAL3=2.045, NN=33,
XL=0.,XR=10.,YB=0.,YT=240.,
$

```

## PROGRAM LISTING (Concluded)

```
$INPUT
X=1.39,1.58.,1.70,1.89,1.96,2.08,2.21,2.46,2.84,3.03,3.16,3.48,3.60,
3.79,4.11,4.43,4.46,4.62,4.74,4.87,5.06,5.69,5.82,6.07,6.45,6.96,
Y=75.,71.,50.,116.,94.,97.,105.,106.,114.,121.,121.,124.,139.,130.,
136.,148.,149.,155.,151.,160.,158.,161.,160.,164.,173.,172., TVAL1=2.064,
TVAL2=2.069, TVAL3=2.074, NN=26,
XL=0.,XR=10.,YB=0.,YT=240.,
$
-PMD,BALE
-FIN
-FIN
```

## DESCRIPTION OF DATA DECK INPUT PARAMETERS

The card immediately following the @XQT card is the first card of the input data deck. This card specifies the number of cases (NCASES) of data that are to be processed. The format is of the form +XX (I3) and appears in columns 1 through 3. The information between the \$INPUT card and the \$ card is associated with a specific set of data and is input under the nonexecutable NAMELIST statement. For example, the input statement in the program is

```
NAMELIST/INPUT/X,Y,TVAL1,TVAL2,TVAL3,NN,XL,XR,YB,YT.
```

The forms the input data take on include variable name and subscripted variable. In the usage above, X and Y are subscripted arrays, and the remaining variables are simple variable names. The specific format of the data can be either integer constants (i.e., +218) or real constants (i.e., 1.85921E+00, with or without the E notation). The description of the variables in the NAMELIST statement follows.

- X            — array containing the values for the independent variable
- Y            — array containing the values for the dependent variable
- TVAL1       — value from t-table for NN-(1 + 1) degrees of freedom
- TVAL2       — value from t-table for NN-(2 + 1) degrees of freedom
- TVAL3       — value from t-table for NN-(3 + 1) degrees of freedom
- NN           — number of data points
- XL           — left plot limit for the horizontal X axis
- XR           — right plot limit for the horizontal X axis
- YB           — bottom plot limit for the vertical Y axis
- YT           — top plot limit for the vertical Y axis

[illegible]

PROGRAMMER COMMENTS:

**OVER**

MICRO FILM		COPIES		COPY FLO		OPER. INIT.
#FILES	#FRAMES	P	F	P	F	SEQ.#

OPERATOR COMMENTS: ☐ SEE TECH. ☐ SEE OPER.

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**OVER**

29

### TYPICAL OUTPUT RESULTS FOR EXAMPLE NO. 2

[illegible]

SEND

## TYPICAL OUTPUT RESULTS FOR EXAMPLE NO. 2 (Continued)

### REGRESSION ANALYSIS APPLICATION TO FILM THICKNESS MEASUREMENT EXPERIMENTS

#### LINEAR, QUADRATIC, AND CUBIC REGRESSION ON INPUT MEASUREMENT DATA

X, INDEPENDENT	Y, DEPENDENT	YC1, DEGREE-1	YC2, DEGREE-2	YC3, DEGREE-3
.38	19.000	16.900	15.516	19.234
.70	22.000	20.646	19.622	21.182
.95	23.000	23.572	22.803	23.058
1.58	32.000	30.948	30.716	28.959
1.70	31.000	32.352	32.207	30.247
1.89	25.000	34.577	34.556	32.377
2.53	40.000	42.069	42.371	40.220
3.03	49.000	47.922	48.370	46.839
3.16	46.000	49.444	49.915	48.600
3.29	53.000	50.966	51.453	50.370
3.35	53.000	51.668	52.161	51.188
3.48	57.000	53.190	53.690	52.963
3.54	54.000	53.892	54.394	53.782
3.67	56.000	55.414	55.914	55.555
3.73	56.000	56.117	56.613	56.371
3.79	53.000	56.819	57.311	57.185
4.05	66.000	59.863	60.321	60.683
4.11	58.000	60.565	61.011	61.481
4.24	61.000	62.087	62.504	63.195
4.36	68.000	63.492	63.876	64.756
4.43	66.000	64.311	64.674	65.656
4.55	73.000	65.716	66.038	67.179
4.62	64.000	66.536	66.830	68.054
4.81	69.000	68.760	68.974	70.376
5.06	69.000	71.686	71.773	73.296
5.19	64.000	73.208	73.220	74.745
5.25	78.000	73.911	73.885	75.396
5.38	78.000	75.433	75.323	76.767
5.44	79.000	76.135	75.984	77.380
6.01	85.000	82.808	82.200	82.512
6.14	84.000	84.330	83.600	83.486
6.26	89.000	85.734	84.888	84.313
6.96	85.000	93.929	92.291	87.597

## TYPICAL OUTPUT RESULTS FOR EXAMPLE NO. 2 (Concluded)

```

--LINEAR REGRESSION COEFFICIENTS
RO= .124512+02  R1= .117066+02  F VALUE= .729913+03
SIGMA Y= .401966+01  SGR0= .699732+00  SGR1= .433306+00

--QUADRATIC REGRESSION COEFFICIENTS
SO= .105907+02  S1= .130312+02  S2= -.185727+00  F VALUE= .361687+03
SIGMA Y= .403972+01  SGR0= .703225+00  SGR1= .164984+01  SGR2= .223129+00
F VALUE FOR SIGNIFICANCE OF X-SQUARE TERM
F21= .919478-03

--CUBIC REGRESSION COEFFICIENTS
TO= .176540+02  T1= .302953+01  T2= .308152+01  T3= -.297834+00  F VALUE= .280993+03
SIGMA Y= .375495+01  SGR0= .653653+00  SGR1= .445316+01  SGR2= .138140+01  SGR3= .124498+00
F VALUE FOR SIGNIFICANCE OF X-CUBE TERM
F32= .656852-02
DETERMINANT VALUE FOR QUADRATIC AND CUBIC REGRESSIONS
DET2= .282084+05  DET3= .256603+08

--INVERSE MATRIX X ORIGINAL MATRIX
QUADRATIC REGRESSION
.100000+01  -.953674+06
-.298023-07  .100000+01

CUBIC REGRESSION
.999989+00  -.305176-04  -.427246-03
.190735-05  .100001+01  .122070-03
-.178814-06  -.476837-06  .999987+00

```

## REFERENCES

1. Junkin, Bobby G.: Regression Analysis Procedures for the Evaluation of Tracking System Measurement Errors. NASA TN D-4826, December 1968.
2. Draper, N. R.; and Smith, H.: Applied Regression Analysis. John Wiley & Sons, Inc., New York, 1967.

## APPROVAL

### APPLICATION OF REGRESSION ANALYSIS TECHNIQUES TO REFRACTORY COATING MEASUREMENT EXPERIMENTS

By Bobby G. Junkin

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A handwritten signature in black ink, appearing to read "H. Hoelzer", is written over a horizontal line.

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